DIGEST Support Document

Part 3 — The ARC System

(The Equal Arc-Second Raster Chart/Map System)

DIGEST Support Document is module-based, and is designed to provide additional information to users of DIGEST Edition 2.1 where required.

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3 The ARC System (The Equal Arc-Second Raster Chart/Map System)

3.1 Scope

This clause defines the Equal Arc-Second Raster Chart/Map System, more usually known as the ARC System. The individual aspects described are:

- the geodetic datum (WGS 84);

- the 16 non-polar zones and the 2 polar zones of the system;

- the system of zone distribution rectangles (ZDRs) for raster images;

- the relative coordinate system used for pixels in the zone distribution rectangles (ARC coordinates being row & column);

- the conversion methods which relate the row & column coordinates to geographical coordinates.

3.2 References

1. Digital Geographic Information Working Group, "Digital Geographic Information Exchange Standards (DIGEST)":

Part 1: General Description; Edition 2.1, September 2000.

Part 2: Theoretical Model, Exchange Structure, and Encapsulation Rules; Edition 2.1, September 2000.

Part 3: Codes and Parameters; Edition 2.1, September 2000.

Part 4: Feature and Attribute Coding Catalogue (FACC); Edition 2.1, September 2000.

2. NIMA Technical Report 8350.2 "Department of Defense World Geodetic System 1984" (Third Edition dated 4 July 1997).

3. MIL-A-89007 "ARC Digitized Raster Graphs (ADRG)"; Edition dated February 1990.

4. Snyder, John P: "Map Projections - A Working Manual" (US Geological Survey Professional Paper 1395, 1987).

3.3 General Description

3.3.1 The ARC System

The Equal Arc-Second Raster Chart/Map system, more usually known as the ARC System, is a special grid system covering the entire ellipsoid of the World Geodetic System 1984. It provides a rectangular coordinate system based on 18 latitudinal zones. These allow an image-dependent relative coordinate system to be used with individual raster images. The design objective of ARC is to provide graphic data in a manner as seamless as possible, and to permit direct display with simple representation of directions.

For the purposes of this description of ARC, there is a defined region of interest called a Distribution Rectangle (DR). More precisely, it is the minimum bounding rectangle around the raster image. It is rectangular with respect to geographical coordinates, so it is defined by two bounding longitudes and two bounding latitudes.

3.3.2 Datum

The geodetic datum is the World Geodetic System 1984. The constants for the WGS 84 ellipsoid (extracted from NIMA TR 8350.2) are:

Semi-major axis: **a** = 6378137 metres (exactly)

Inverse flattening: **1/f** = 298.257223563 (now a defining parameter)
Semi-minor axis: **b** = 6356752.3142 metres (derived from 1/f and a)
Eccentricity squared: $\mathbf{e}^2$ = 0.00669437999013 (derived from 1/f)

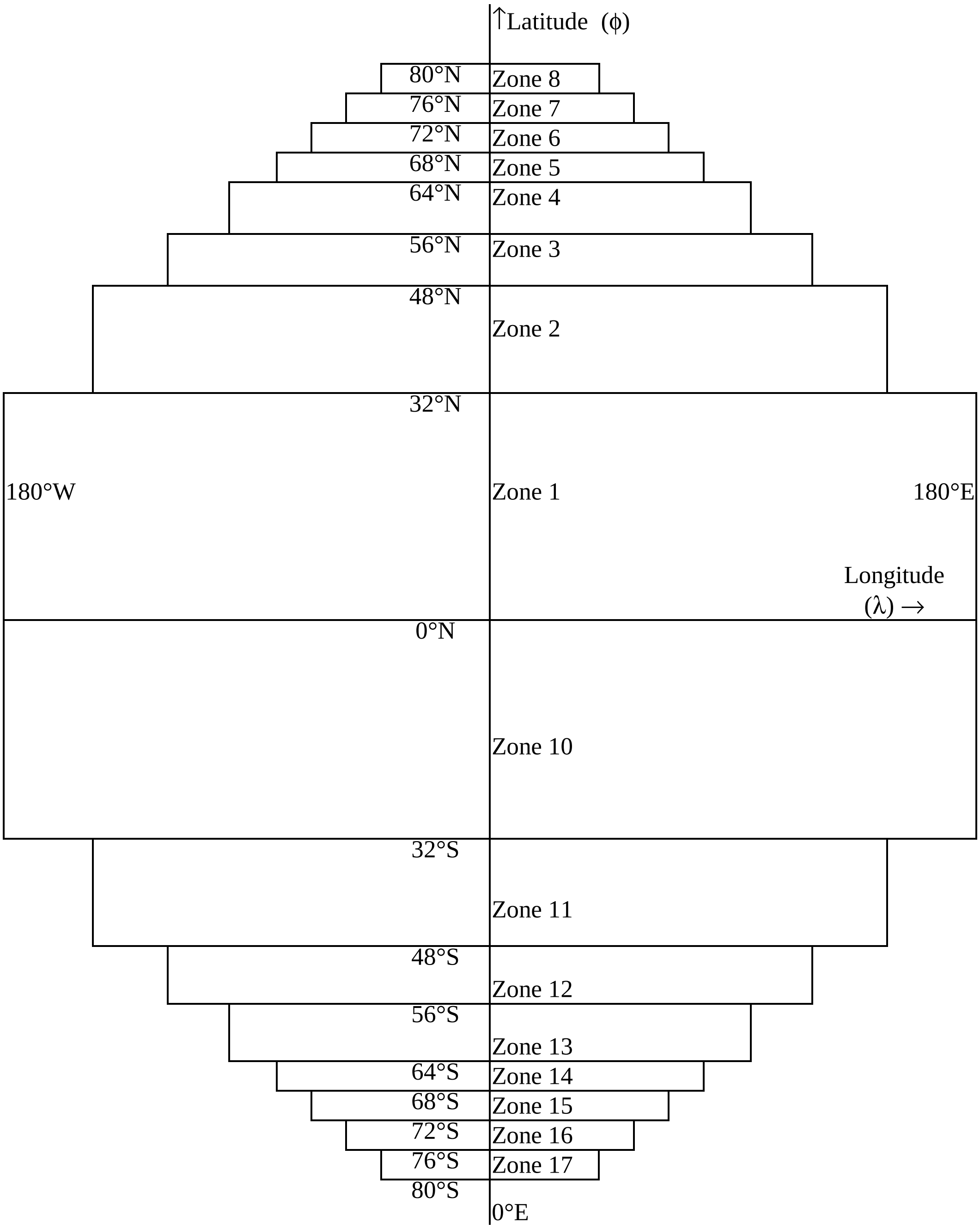
3.3.3 Zones

The basic zones, in the sense of the zones exclusive of overlaps, are defined in Table 3-1 and illustrated in Figures 3-1 and 3-2.

Table 3-1 ARC Zones

Northern Zones	**Latitude Range**	**Southern Zones**	**Latitude Range**
Zone 1	0°N to 32°N	Zone 10	0°S to 32S°
Zone 2	32°N to 48°N	Zone 11	32°S to 48S°
Zone 3	48°N to 56°N	Zone 12	48°S to 56S°
Zone 4	56°N to 64°N	Zone 13	56°S to 64S°
Zone 5	64°N to 68°N	Zone 14	64°S to 68S°
Zone 6	68°N to 72°N	Zone 15	68°S to 72S°
Zone 7	72°N to 76°N	Zone 16	72°S to 76S°
Zone 8	76°N to 80°N	Zone 17	76°S to 80S°
Zone 9 (Polar)	80°N to 90°N	Zone 18 (Polar)	80°S to 90S°

Notes:
1. These ranges are exclusive of overlaps.
2. All zones include the full longitude range.



This illustrates the relative size of non-polar zones, each of which represents the range 180°W to 180°E.

Figure 3-1. ARC System Non-Polar Zones

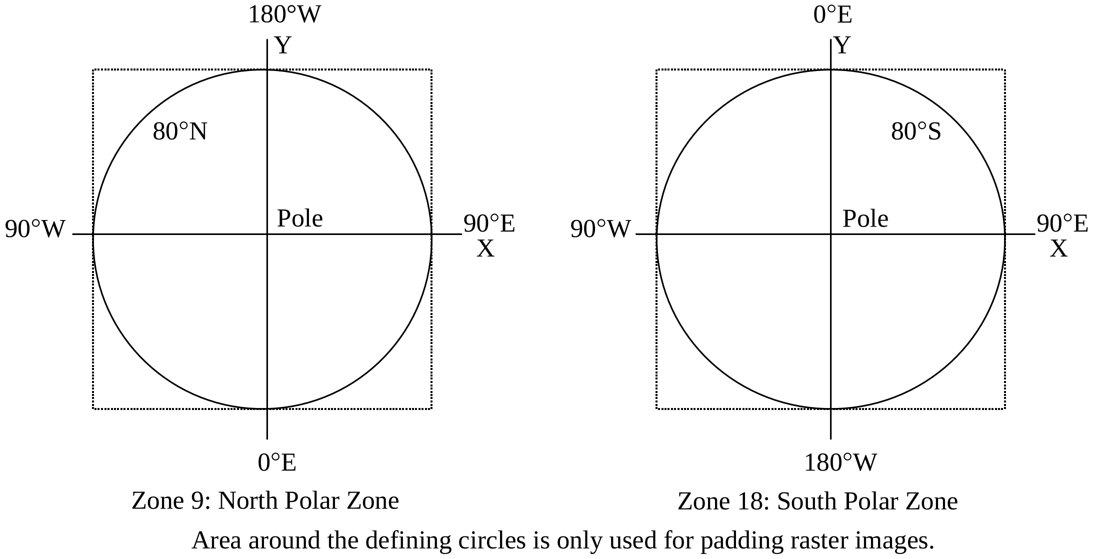


Figure 3-2. ARC System Polar Zones

For raster images based on the ARC System, each non-polar zone is considered to have a Zone Overlap. This is the overlap into the next zone poleward by 1024 rows of pixels, thus duplicating coverage of 1024 rows of the next zone. The precise north-south distance covered by the overlap depends on the scale of data portrayed in that zone.

At latitudes defined by the ARC system zone limits, the raster data for a DR is divided into images called Zone Distribution Rectangles (ZDRs). There is one ZDR for each ARC system zone (including the overlap) covering any part of the DR; the ZDR depicts all the DR data that falls within that zone's limits. A DR covered by only one zone will thus contain a single ZDR; a DR covered by more than one zone will contain more than one ZDR. Note that a DR which is fully covered by one zone but which lies at the equatorward limit of that zone is also covered by the overlap portion of the adjoining equatorward zone and will contain 2 ZDRs (one depicting the entire DR and a second depicting that part of the DR covered by the adjoining zone's overlap).

Another concept related to ARC zones is the Standard Latitude. It is best understood as the latitude chosen for distortion-free representation of east-west scale. The circumference of the standard latitude in a non-polar zone is treated as the length of the band representing that zone.

3.3.4 Theoretical Grid System

The grid of each ARC zone is theoretical, in the sense that the zone eastings and northings are not the same as ARC coordinates. ARC coordinates are actually relative coordinates, referenced to the ZDR, scaled from the zone eastings and northings.

3.3.4.1 Grid Of Each Non-Polar Zone

The projection used is the Equirectangular projection described in Snyder, Pages 90-91. The formulae are

$$x = (R \cos \phi_{std})\, \lambda \quad \text{and} \quad y = R\, \phi\,,$$

where ϕ and λ are in radians, R is the radius of the approximating sphere and ϕ_{std} is the standard latitude.

The standard latitude is an intermediate latitude between the two limits, defined in such a way as to make the maximum stretch equivalent to the maximum shrink. The precise definition is given in 3.5. Table 3-2 gives the values for each zone.

Table 3-2 Standard Latitudes Of The ARC Non-Polar Zones

Northern Zones	**Standard Latitude**	**Southern Zones**	**Standard Latitude**
Zone 1	22.94791772°N	Zone 10	22.94791772S°
Zone 2	41.12682127°N	Zone 11	41.12682127S°
Zone 3	52.28859923°N	Zone 12	52.28859923S°
Zone 4	60.32378942°N	Zone 13	60.32378942S°
Zone 5	66.09421768°N	Zone 14	66.09421768S°
Zone 6	70.10896259°N	Zone 15	70.10896259S°
Zone 7	74.13230145°N	Zone 16	74.13230145S°
Zone 8	78.17283750°N	Zone 17	78.17283750S°

Although the maximum distortion in each zone (without overlap) is minimised, it is still very large in comparison to most grid systems. In zone 2, for example, there is a $1 \rightarrow 1.1253$ stretch along the Pole-ward latitude and a $1.1253 \rightarrow 1$ (or $1 \rightarrow 0.8887$) shrink along the Equatorward latitude. The distortion, which can be up to 18% between the formal latitude limits can rise to 25% in the overlap, although this depends on the size of the overlap and how near it is to the pole.

The absolute coordinates (x, y) on the theoretical grid are not used in practice, because it is relatively easy to convert directly between WGS 84 geographical coordinates (ϕ, λ) and ARC coordinates (r, c): r is a linear function of ϕ, and c is a linear function of λ. Figure 3-1 therefore omits the X axis (which would be the line representing the Equator) and the Y axis (which would be the line representing the zero meridian).

3.3.4.2 Grid Of Each Polar Zone

The projection used is the Azimuthal Equidistant projection, polar aspect, spherical form, described in Snyder, Pages 191-195. Its simplicity is best understood by looking at Figure 3-2 and imagining each 80° polar cap represented as a flat circle with each degree of latitude treated as one tenth of the radius. (Each arc-second of latitude is represented as 1/36000 of the radius.)

Given that the ARC System applies the projection with zero as the central longitude, the X and Y axes are as shown in Figure 3-2. The projection formulae are

$$x = (R \sin \lambda)\, (\pi/2 - \phi) \quad \text{and} \quad y = (-R \cos \lambda)\, (\pi/2 - \phi)$$

in the north polar aspect, and

$$x = (R \sin \lambda)(\pi/2 + \phi) \text{ and } \qquad y = (R \cos \lambda)(\pi/2 + \phi)$$

in the south polar aspect. In each case, ϕ and λ are in radians, while R is the radius of the approximating sphere. ARC uses the semi-major axis 6378137m as R.

The absolute coordinates (x, y) on the theoretical grid are only used as intermediate values when converting between WGS 84 geographical coordinates (ϕ, λ) and ARC coordinates (r, c).

Distortion is considerably less than in the non-polar zones. Both east-west and north-south distortion are less than 1%, and the same is true of one relative to the other.

3.3.5 ARC Coordinates

ARC coordinates (r, c) specify the row and column of a pixel in a ZDR. For that reason, they are also known as ZDR coordinates. The top left pixel has ZDR coordinates (0, 0) and the position of its top-left corner is denoted (ϕ_0, λ_0) in WGS 84 coordinates. See Figure 3-3.

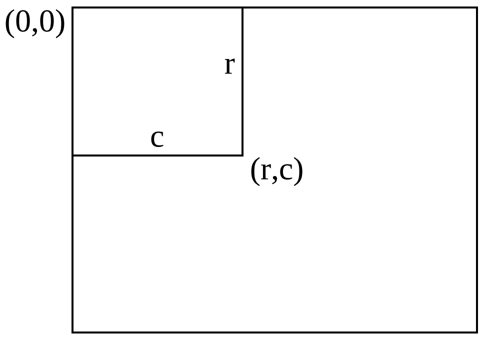


Figure 3-3. ARC Coordinates Within A ZDR

3.3.5.1 Pixel Spacing

Since r and c are the number of pixels from the top-left corner to a given position, their value depends on the size of the pixel.

The constant B_s is used to denote the number of pixels per 360° of latitude. Fixing that number is equivalent to fixing the height of a pixel. Its value depends on the scale used for the raster image, hence the subscript s, although it must be a multiple of 512. The precise relationship is given in 3.4.6, but is best illustrated by Table 3-3.

Table 3-3 Examples of North-South Pixel Spacing Constants

Scale→	1:1 000 000	1:500 000	1:250 000	1:100 000	1:50 000
B_s in any zone	400 384	800 768	1 601 536	4 003 840	8 007 680

In the non-polar zones, the constant A_{sz} is used to denote the number of pixels needed go round the parallel of standard latitude, ie the number of pixels per 360° of longitude. Fixing that number is equivalent to fixing the width of a pixel. Its value depends not only on scale but on the zone; this because the parallels are shorter in the zones nearer the poles. Again, it must be a multiple of 512. The precise relationship is given in 3.4.6, but is best illustrated by the following examples:

Table 3-4 Examples of East-West Pixel Spacing Constants

Scale→	1:1 000 000	1:500 000	1:250 000	1:100 000	1:50 000
A_{sz} in zones 1, 10	369 664	739 328	1 478 656	3 696 640	7 393 280
A_{sz} in zones 2, 11	302 592	615 184	1 210 368	3 025 920	6 151 840
A_{sz} in zones 3, 12	245 760	491 520	983 040	2 457 600	4 915 200
A_{sz} in zones 4, 13	199 168	398 336	796 672	1 991 680	3 983 360
A_{sz} in zones 5, 14	163 328	326 656	653 312	1 633 280	3 266 560
A_{sz} in zones 6, 15	137 216	274 432	548 864	1 372 160	2 744 320
A_{sz} in zones 7, 16	110 080	220 160	440 320	1 100 800	2 201 600
A_{sz} in zones 8, 17	82 432	164 864	329 728	824 320	1 648 640

In the polar zones, only the constant B_s is really needed. However, to preserve the association of A_{sz} and B_s with pixel-width and pixel-height respectively, it is convenient to specify that

> A_{sz} (in the polar case) is the number of pixels needed to go round the 90°N-90°S meridian circumference (making it the pixel-spacing constant along the X axis), and
>
> B_s is the number of pixels needed to go round the 0°N-180°N meridian circumference (making it the pixel-spacing constant along the Y axis).

In polar zones, A_{sz} is always equal to B_s, and B_s has already been tabulated for all zones.

3.3.5.2 Possible Modification Of Image Origin

When using the ARC System, there may be a requirement to have the point (ϕ_0, λ_0) an exact number of tiles from the zero meridian and the Equator. The tile could, for instance, be 128 by 128 pixels. An algorithm for obtaining such a point (ϕ_0, λ_0) from an original upper-left corner (ϕ_{UL}, λ_{UL}) is given in 3.4.5.

3.3.6 Coordinate Relationships

The relationship between the WGS 84 geographical coordinates (ϕ, λ) and the ARC coordinates (r, c) depend on:

> the position (ϕ_0, λ_0) of the top-left corner of the ZDR, and
>
> the pixel-spacing constants A_{sz} and B_s, which in turn depend on scale and zone.

The formulae are given in 3.4.3 and 3.4.4. In the non-polar zones, r is a rounded negative multiple of ($\phi - \phi_0$) while c is a rounded positive multiple of ($\lambda - \lambda_0$). In the polar zones, it is easier to relate (ϕ, λ) to the absolute grid coordinates (x, y).

3.4 ARC Coordinate Conversions

This sub-clause is not essential for a general understanding of the ARC System. However, it is normative in the sense of describing the formulae needed for implementation.

3.4.1 Notation Conventions

Throughout sub-clause 3.4, *round* ( ) denotes "nearest integer to".

Throughout sub-clause 3.4, ϕ_{84} denotes latitude with respect to WGS 84 and λ_{84} denotes longitude with respect to WGS 84. (The subscripts are not strictly necessary in a paper describing the relationship between ARC coordinates and WGS 84 coordinates. However, the subscripts help to avoid ambiguity, particularly if the information is applied to a raster specification where a datum of a source graphic is involved as well as the WGS 84 datum.)

3.4.2 ARC System Parameters

Parameter	Description	Tags
(ϕ_0, λ_0)	WGS 84 coordinates of the Zone Distribution Rectangle (ZDR) (0, 0) pixel	PSO, LSO
$\mathbf{A_{sz}}$	East-West pixel spacing at scale 1:S in zone Z	ARV
$\mathbf{B_s}$	North-South pixel spacing at scale 1:S	BRV

where units for latitude and longitude are decimal degrees.

Note that other parameters would be needed if the datum of the source graphic is involved in the raster application.

3.4.3 Latitude and Longitude (ϕ_{84}, λ_{84}) of a Pixel at (r, c)

Parameters Used:

Parameter	Description	Tag
(ϕ_0, λ_0)	WGS 84 coordinates of the ZDR (0, 0) pixel	PSO, LSO
$\mathbf{A_{sz}}$	East-West pixel spacing at scale 1:S in zone Z	ARV
$\mathbf{B_s}$	North-South pixel spacing at scale 1:S	BRV

3.4.3.1 Conversion of (r,c) to (ϕ_{84}, λ_{84}) in a Non-Polar Zone

$$\phi_{84} = \phi_0 - (360°r/\mathbf{B_s})$$
$$\lambda_{84} = \lambda_0 + (360°c/\mathbf{A_{sz}})$$

3.4.3.2 Conversion of (r,c) to (ϕ_{84}, λ_{84}) in North Polar Zone

(1) ARC System Coordinates (x_0, y_0) at (ϕ_0, λ_0)

$$x_0 = (\mathbf{B_s}/360°)\,(90° - \phi_0)\,\sin(\lambda_0)$$
$$y_0 = -\,(\mathbf{B_s}/360°)\,(90° - \phi_0)\,\cos(\lambda_0)$$

(2) ARC System Coordinates (x, y) at (r, c)

$x = x_0 + c$
$y = y_0 - r$

(3) WGS 84 Coordinates (ϕ_{84}, λ_{84}) at (x, y)

$\phi_{84} = 90° - [(x^2 + y^2)^{1/2} / (\mathbf{B_s}/360°)]$
$\lambda_{84} = \arccos[-y/(x^2 + y^2)^{1/2}]$ if $x > 0$ or $x = 0, y \neq 0$
$\lambda_{84} = -\arccos[-y/(x^2 + y^2)^{1/2}]$ if $x < 0$
$\lambda_{84} = 0°$ (i.e., undefined) if $x = y = 0$

where $0° <= \arccos[-y/(x^2 + y^2)^{1/2}] <= 180°$

3.4.3.3 Conversion of (r,c) to (ϕ_{84}, λ_{84}) in South Polar Zone

(1) ARC System Coordinates (x_0, y_0) at (ϕ_0, λ_0)

$x_0 = (\mathbf{B_s}/360°)(90° + \phi_0)\sin(\lambda_0)$
$y_0 = (\mathbf{B_s}/360°)(90° + \phi_0)\cos(\lambda_0)$

(2) ARC System Coordinates (x, y) at (r, c)

$x = x_0 + c$
$y = y_0 - r$

(3) WGS 84 Coordinates (ϕ_{84}, λ_{84}) at (x, y)

$\phi_{84} = -90° + [(x^2 + y^2)^{1/2} / (\mathbf{B_s}/360°)]$
$\lambda_{84} = \arccos[y/(x^2 + y^2)^{1/2}]$ if $x > 0$ or $x = 0, y \neq 0$
$\lambda_{84} = -\arccos[y/(x^2 + y^2)^{1/2}]$ if $x < 0$
$\lambda_{84} = 0°$ (i.e., undefined) if $x = y = 0$

where $0° <= \arccos[y/(x^2 + y^2)^{1/2}] <= 180°$

3.4.4 ZDR Pixel Coordinates (r, c) of a Geographic Point (ϕ_{84}, λ_{84})

Parameters Used:

Parameter	Description	Tag
(ϕ_0, λ_0)	WGS 84 coordinates of the ZDR (0, 0) pixel	PSO, LSO
$\mathbf{A_{sz}}$	East-West pixel spacing at scale 1:S in zone Z	ARV
$\mathbf{B_s}$	North-South pixel spacing at scale 1:S	BRV

3.4.4.1 Conversion of (ϕ_{84}, λ_{84}) to (r,c) in a Non-Polar Zone

$$r = round\,[(\phi_0 - \phi_{84})\,(\mathbf{B_s}/360°)]$$
$$c = round\,[(\text{abs}\,(\lambda_{84} - \lambda_0) \bmod 360°)\,(\mathbf{A_{sz}}/360°)]$$

3.4.4.2 Conversion of (ϕ_{84}, λ_{84}) to (r,c) in North Polar Zone

(1) ARC System Coordinates (x_0, y_0) at (ϕ_0, λ_0)

$$x_0 = (\mathbf{B_s}/360°)\,(90° - \phi_0)\,\sin(\lambda_0)$$
$$y_0 = -\,(\mathbf{B_s}/360°)\,(90° - \phi_0)\,\cos(\lambda_0)$$

(2) ARC System Coordinates (x, y) at $(\phi_{84}, \lambda_{84})$

$$x = (\mathbf{B_s}/360°)\,(90° - \phi_{84})\,\sin(\lambda_{84})$$
$$y = -\,(\mathbf{B_s}/360°)\,(90° - \phi_{84})\,\cos(\lambda_{84})$$

(3) ZDR Pixel Coordinates (r, c) at (x, y)

$$r = round\,(y_0 - y)$$
$$c = round\,(x - x_0)$$

3.4.4.3 Conversion of (ϕ_{84}, λ_{84}) to (r,c) in South Polar Zone

(1) ARC System Coordinates (x_0, y_0) at (ϕ_0, λ_0)

$$x_0 = (\mathbf{B_s}/360°)\,(90° + \phi_0)\,\sin(\lambda_0)$$
$$y_0 = (\mathbf{B_s}/360°)\,(90° + \phi_0)\,\cos(\lambda_0)$$

(2) ARC System Coordinates (x, y) at $(\phi_{84}, \lambda_{84})$

$$x = (\mathbf{B_s}/360°)\,(90° + \phi_{84})\,\sin(\lambda_{84})$$
$$y = (\mathbf{B_s}/360°)\,(90° + \phi_{84})\,\cos(\lambda_{84})$$

(3) ZDR Pixel Coordinates (r, c) at (x, y)

$$r = round\,(y_0 - y)$$
$$c = round\,(x - x_0)$$

3.4.5 Computation of Coordinates ϕ_0 and λ_0 (For Alignment of Tiling Scheme)

This algorithm is applicable when there is a requirement to have the point (ϕ_0, λ_0) an exact number of tiles from the zero meridian and the Equator, given a tile-size of 128×128 pixels.

Parameters Used:

Parameter	Description	Tag
$\mathbf{A_{sz}}$	East-West pixel spacing at scale 1:S in zone Z	ARV

$\mathbf{B_s}$	North-South pixel spacing at scale 1:S	BRV

The coordinates of the image origin (ϕ_0, λ_0) depend on the pixel spacing constants adjusted for the scale and zone of the image data.

The width of the pixel is 360°/$\mathbf{A_{sz}}$ and in the E-W direction and 360°/$\mathbf{B_s}$ in the N-S direction. For polar zones, $\mathbf{A_{sz}} = \mathbf{B_s}$.

Let (ϕ_{UL}, λ_{UL}) denote the WGS 84 coordinates of the upper left point of the image. For non-polar zones, ϕ_{UL} is the northernmost latitude and λ_{UL} is the westernmost longitude.

3.4.5.1 (ϕ_0, λ_0) within a Non-Polar Zone

Let $\Delta\phi$ and $\Delta\lambda$ denote the length of a tile of 128 by 128 pixels.

$$\Delta\phi = 46080°/\mathbf{B_s}$$
$$\Delta\lambda = 46080°/\mathbf{A_{sz}}$$

ϕ_0 is ϕ_{UL} rounded <u>up</u> to the next integral multiple of $\Delta\phi$ (to ensure an exact number of tiles to the Equator). Equivalently, ϕ_0 is n($\Delta\phi$) where n is $\phi_{UL}/\Delta\phi$ rounded up to an integer.

λ_0 is λ_{UL} rounded <u>down</u> to the next integral multiple of $\Delta\lambda$ (to ensure an exact number of tiles to the Prime Meridian). Equivalently, λ_0 is n($\Delta\lambda$) where n is $\lambda_{UL}/\Delta\lambda$ rounded down to an integer.

3.4.5.2 (ϕ_0, λ_0) within North Polar Zone

$x_0 = (\mathbf{B_s}/360°)\,(90° - \phi_{UL})\,\sin(\lambda_{UL})$	rounded down to the next multiple of 128.
$y_0 = -(\mathbf{B_s}/360°)\,(90° - \phi_{UL})\,\cos(\lambda_{UL})$	rounded up to the next multiple of 128.
$\phi_0 = 90° - [(x_0^2 + y_0^2)^{1/2}\,(360°/\mathbf{B_s})]$	
$\lambda_0 = \arccos[-y_0/(x_0^2 + y_0^2)^{1/2}]$	if $x_0 > 0$ or $x_0 = 0$, $y_0 \neq 0$
$\lambda_0 = -\arccos[-y_0/(x_0^2 + y_0^2)^{1/2}]$	if $x_0 < 0$
$\lambda_0 = 0°$	if $x_0 = y_0 = 0$

where the range of arccos is 0° to 180°.

3.4.5.3 (ϕ_0, λ_0) within South Polar Zone

$x_0 = (\mathbf{B_s}/360°)\,(90° + \phi_{UL})\,\sin(\lambda_{UL})$	rounded down to the next multiple of 128.
$y_0 = (\mathbf{B_s}/360°)\,(90° + \phi_{UL})\,\cos(\lambda_{UL})$	rounded up to the next multiple of 128.
$\phi_0 = -90° + [(x_0^2 + y_0^2)^{1/2}\,(360°/\mathbf{B_s})]$	
$\lambda_0 = \arccos[y_0/(x_0^2 + y_0^2)^{1/2}]$	if $x_0 > 0$ or $x_0 = 0$, $y_0 \neq 0$
$\lambda_0 = -\arccos[y_0/(x_0^2 + y_0^2)^{1/2}]$	if $x_0 < 0$
$\lambda_0 = 0°$	if $x_0 = y_0 = 0$

where the range of arccos is 0° to 180°.

3.4.6 Computation of Constants A_{sz} and B_s

Pixel spacing constants and spacing intervals for all zones at the scale 1:1,000,000 are shown in Table 3-5 below.

Table 3-5 Pixel Spacing Constants **A** and **B**

	Zone Limits [1]				Nominal Pixel Spacing [2]	
Zone Number	Equator -ward	Pole -ward	**A**	**B**	Longitude (microns)	Latitude (microns)
1, 10	0	32	369664	400384	99.9	99.9
2, 11	32	48	302592	400384	99.9	99.9
3, 12	48	56	245760	400384	100.0	99.9
4, 13	56	64	199168	400384	99.9	99.9
5, 14	64	68	163328	400384	99.7	99.9
6, 15	68	72	137216	400384	99.7	99.9
7, 16	72	76	110080	400384	99.8	99.9
8,17	76	80	82432	400384	100.0	99.9
9,18	80	90	400384	400384	99.9	99.9

NOTES:
1. Latitudes are shown unsigned for convenience.
2. Measured at the latitude which gives equivalent stretch and shrink at zone limits on the WGS 84 ellipsoid.

To compute the pixel spacing constants for a graphic at scale 1:S in zone Z (i.e., $\mathbf{A_{sz}}$ and $\mathbf{B_s}$), first compute the real scale factor **N** using the following equation:

$$\mathbf{N} = 1{,}000{,}000 / S$$

The **A** and **B** values for 1:1,000,000 are multiplied by **N** and the results rounded up to the next multiple of 512 to give the values $\mathbf{A_{sz}}$ and $\mathbf{B_s}$.

NOTE: The **A** and **B** values provided with the image data may already be adjusted for the scale and zone of the image data.

3.5 Distortion Analysis

Distortion in the ARC System is significant in the non-polar zones, where it is almost entirely due to distortion in the east-west directions. Distortion can be quantified in terms of the latitude, the ellipsoidal constant e^2, and (in the east-west case) the zone.

3.5.1 North-South Distortion

As shown in Table 3-6, distortion in the north-south directions is small in all zones. The effect of treating meridians as circles with a 6378137m radius is to treat 1″ of latitude as 30.9221m. This is only a 0.34% shrink of the true value at the poles (31.0261m) and only a 0.67% stretch of the true value at the Equator (30.7151m). The north-south distortion ratio (projected distance divided by ellipsoidal distance) at latitude ϕ is

$(1 - e^2\sin^2\phi)^{3/2} / (1 - e^2)$.

Table 3-6 North-South Distortion at Zone-Limit Latitudes

Latitude (unsigned)	**N-S Distortion Factor**	**Shrink (%)**	**Latitude (unsigned)**	**N-S Distortion Factor**	**Stretch (%)**
90°	0.996647	0.34	48°	1.001162	0.12
80°	0.996951	0.30	32°	1.003902	0.39
76°	0.997237	0.28	0°	1.006739	0.67
72°	0.997609	0.24			
68°	0.998061	0.19			
64°	0.998584	0.14			
56°	0.999799	0.02			

3.5.2 East-West Distortion

East-West distortion is so much greater than north-south distortion (except in the polar zones where both are small) that it is usual to quantify distortion purely in terms of the east-west direction. The maximum stretch and shrink for each zone is shown in Table 3-7 below. Also indicated is the standard latitude at which nominal pixel spacing is defined. These values apply for all scales of data.

Table 3-7 Standard Latitudes and Maximum Distortion (Exclusive of Overlap)

Zone	Latitude (degrees)			Distortion	
	Equator-ward	Standard (zero distortion)	Pole-ward	Maximum Stretch (%)	Maximum Shrink (%)
1, 10	0	22.94791772	32	8.54	7.87
2, 11	32	41.12682127	48	12.53	11.13
3, 12	48	52.28859923	56	9.36	8.56
4, 13	56	60.32378942	64	12.92	11.44
5, 14	64	66.09421768	68	8.17	7.55
6, 15	68	70.10896259	72	10.09	9.17
7, 16	72	74.13230145	76	13.01	11.51
8, 17	76	78.17283750	80	18.03	15.27
9, 18	80	81.94975621	90	0.18	0.34

The mathematics of the east-west distortion is given below.

(1) In the non-polar zones, to minimise the maximum distortion in the zone, the standard latitude ϕ_{std} is chosen such that its circumference is the geometric mean of the two bounding circumferences of the zone. Equivalently, ϕ_{std} is the value of ϕ for which

$$\log \cos \phi \; - \; 0.5\log(1 - e^2\sin^2\phi)$$

is halfway between its values at the bounding latitudes. The east-west distortion ratio (projected distance divided by ellipsoidal distance) at latitude ϕ is

$$[\cos \phi \sqrt{(1 - e^2\sin^2\phi_{std})}]/[\cos \phi_{std} \sqrt{(1 - e^2\sin^2\phi)}].$$

This formula can be used to compute the east-west distortion in the overlap of a non-polar zone.

(2) In the polar zones, the standard latitude is not a defining parameter of the projection, but exists in the sense that there is a latitude of zero distortion. The east-west distortion ratio (projected distance divided by ellipsoidal distance) at latitude ϕ is

$$(\theta/\sin\theta)\sqrt{(1 - e^2\sin^2\phi)},$$

where θ is the difference in radians between ϕ and the latitude of the pole. The standard latitude is the latitude where that ratio equals 1, and that latitude is 81.94975621°.